

**Response models and minimal designs for mixtures of n of m items
useful for intercropping and other investigations**

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SUMMARY

Consider the situation where n of m cultivars are grown together in a mixture such as that found in intercropping investigations. Response model equations for $n=2$ are formulated in a manner akin to that found for a diallel crossing experiment in genetics. This situation varies considerably from diallel crossing in that $n \geq 2$ and yields may or may not be available for each member of the mixture. Response model equations were formulated and then minimal treatment designs were obtained to derive least squares solutions for the parameters of the model. This was done for both cases, i.e., when individual member yields were available and when they were not. Variances of estimable contrasts are also given. Applications to other areas of investigation with examples are given.

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1. INTRODUCTION

Intercropping investigations involve the growing of two or more crops on the same area of land either simultaneously, partially at the same time, or sequentially. It is a centuries-old practice in tropical agriculture, and to some extent in temperate zone agriculture. Agricultural, biological, and statistical investigations have tended to ignore the problems of research in this area due to the complexities involved in modeling and interpreting such experiments.

Consider an investigation involving m cultivars, say $\{1, 2, \dots, m\}$. One can form v non-empty sets S_1, S_2, \dots, S_v of the m cultivars. These sets may all be of equal or unequal sizes and they may be of any cardinality $1, 2, \dots, m$. The class of sets so formed is called the treatment design of the m cultivars and these v treatments will be used in an appropriate experiment design like a completely randomized design, a randomized complete block design, an incomplete block design, etc. If the set S_α has a single element i , then $S_\alpha = \{i\}$ is called the i^{th} sole cultivar or uni-blend. If $S_\alpha = \{i, j\}$, then the treatment S_α is called the bi-blend of cultivars i and j . If $S_\alpha = \{i, j, k\}$ then the treatment S_α is called the tri-blend of cultivars i , j , and k . If $S_\alpha = \{i_1, i_2, \dots, i_n\}$, the treatment S_α is then called the n -blend of cultivars i_1, i_2, \dots, i_n .

For simplicity, let us consider the experiment design used with the v treatments to be an orthogonal design. The data can be collected either for each individual cultivar of the treatment S_α associated with an experimental unit, or the data can be collected for the whole experimental unit to which the treatment S_α is applied. Let $S_\alpha = \{i_1, i_2, \dots, i_n\}$. In the former case, let $\bar{y}_{i_j}(S_\alpha)$ denote the mean yield of the i_j cultivar used in

the treatment S_α for $j=1,2,\dots,n$. In the latter case, let \bar{y}_{S_α} denote the mean yield of the treatment S_α . Note that in \bar{y}_{S_α} only the total of n cultivars is available and the individual cultivar yields are not. If more complicated experiment designs are used for the v treatments, one may replace $\bar{y}_{i_j(S_\alpha)}$ and \bar{y}_{S_α} by the estimated adjusted treatment effects appropriate to the experiment design used. Thus without loss of generality we assume the response variable $\bar{y}_{i_j(S_\alpha)}$ or \bar{y}_{S_α} for further analysis discussed in this paper.

In Section 2 we develop a model to interpret $\bar{y}_{i_j(S_\alpha)}$ and give minimal designs which will enable us to estimate all the parametric contrasts of interest. In Section 3, an analysis of such minimal designs will be presented. In Sections 4 and 5 a similar approach will be used for \bar{y}_{S_α} . In Section 6, variances for various contrasts are given. In the concluding section we will give some applications and remarks.

2. MODEL AND MINIMAL DESIGNS WITH RESPONSE VARIABLE $\bar{y}_{i_j(S_\alpha)}$

When we consider the response of a cultivar i_j used in the treatment $S_\alpha = \{i_1, i_2, \dots, i_n\}$, it is affected by the following components:

(i) its relative performance as a monoculture, and such an effect will be denoted by $\mu + \tau_{i_j}^*$;

(ii) Its effect because of its use in mixtures, and such an effect will be called general mixing ability. It will be denoted by δ_{i_j} ;

(iii) its ability to respond well (or poorly) because of the presence of each of the other $n-1$ cultivars in that blend. We denote such effects by $\gamma_{i_j(i_k)}$ for $k = 1, 2, \dots, n; k \neq j$ and we call them the first order specific mixing abilities of the i_j cultivar with the i_k cultivar. One can

easily note that $\gamma_{i_j(i_k)} \neq \gamma_{i_k(i_j)}$ and thus the first order mixing abilities are not the usual first order interaction of the cultivars i_j and i_k ;

(iv) its ability to respond well (or poorly) because of the presence of each distinct pair of the other $n-1$ cultivars in that blend. We denote such effects by $\gamma_{i_j(i_k, i_l)}$ for $k, l = 1, 2, \dots, n$; $k \neq j$, $l \neq j$, $k < l$, and they can be called second order specific mixing abilities of the i_j cultivar with i_k and i_l cultivars; and

(v) Continuing in this fashion, it depends on the third, fourth, $\dots, (n-1)^{th}$ order specific mixing abilities of the i_j cultivar with the other cultivars.

As an illustration, it can be noted that

$$E\left(\bar{y}_{i_1}(S_\alpha)\right) = \mu + \tau_{i_1}^* + \delta_{i_1} + \sum_{k=2}^n \gamma_{i_1(i_k)} + \sum_{\substack{k, l=2 \\ k < l}}^n \gamma_{i_1(i_k, i_l)} \\ + \dots + \gamma_{i_1(i_2, i_3, \dots, i_n)},$$

where $E(\cdot)$ is the expected value of the random variable in the parentheses.

In an experiment involving m cultivars, it is possible for each cultivar to have a monoculture effect, general mixing ability, specific mixing abilities of first, second, \dots , and $(m-1)^{th}$ order. In practice, an experimenter may be interested in drawing inferences on at most t^{th} order specific mixing abilities for a given t ($t \leq m-1$). By using uni-blends, bi-blends, \dots , $(t+1)$ -blends, one can make such inferences. When using mixtures with different number of cultivars, the problems of plant density per hectare for each cultivar in the mixture and of unequal error variances arise. It becomes absolutely necessary to use uni-blends if the

experimenter is interested in drawing conclusions about general mixing abilities. If uni-blends are not used in the experiment, τ_{ij}^* and δ_{ij} effects will be completely confounded, and one can draw inferences only about $\tau_{ij} = \tau_{ij}^* + \delta_{ij}$. In this paper, we restrict our attention to blends using the same number of cultivars, that is, the treatment sets S_α having the same cardinality.

The effects can be reparameterized, if necessary, and the following conditions or restrictions may be imposed on the parameters:

$$\sum_{i=1}^m \tau_i = 0 ,$$

$$\sum_{\substack{j_1=1 \\ j_1 \neq i}}^m \gamma_{i(j_1)} = 0 , \text{ for a given } i,$$

$$\sum_{\substack{j_2=1 \\ j_2 \neq i, j_1}}^m \gamma_{i(j_1, j_2)} = 0 , \text{ for a given } i \text{ and } j_1 (i \neq j_1),$$

$$\vdots$$

$$\sum_{\substack{j_t=1 \\ j_t \neq i, j_1, j_2, \dots, j_{t-1}}}^m \gamma_{i(j_1, j_2, \dots, j_t)} = 0 , \text{ for a given } i \text{ and } j_1, j_2, \dots, j_{t-1}$$

$$(j_1 < j_2 < \dots < j_{t-1},$$

$$i \neq j_1, j_2, \dots, j_{t-1}) .$$

For any k , the number of k^{th} order specific combining ability parameters on the i^{th} cultivar, $\gamma_{i(j_1, j_2, \dots, j_k)}$ is $\binom{m-1}{k}$, and the number of restrictions is $\binom{m-1}{k-1}$. Thus, the number of independent k^{th} order specific combining ability parameters on the i^{th} cultivar is

$$\binom{m-1}{k} - \binom{m-1}{k-1} = \frac{(m-1)!}{k!(m-k)!} (m-2k) ,$$

and hence the total number of k^{th} order specific combining ability parameters is

$$m \frac{(m-1)!}{k!(m-k)!} (m-2k) = \binom{m}{k} (m-2k) .$$

In view of this, to draw inferences about τ_i and at most t^{th} order specific mixing abilities of the cultivars, the number of responses needed is

$$1 + (m-1) + \binom{m}{1} (m-2) + \binom{m}{2} (m-4) + \dots + \binom{m}{t} (m-2t) = \binom{m}{t+1} (t+1),$$

for the mean, τ_i s (cultivar effects in a mixture), and first to t^{th} order specific mixing abilities, respectively.

Thus using m cultivars, one can draw inferences for at most $[m/2-1]^{\text{th}}$ order specific mixing abilities where $[\cdot]$ is the greatest integer function because $m - 2t > 0$. On the other hand, if an experimenter is interested in estimating t^{th} order specific mixing abilities, the number of cultivars to be used in the experiment m should satisfy $m > 2t$. The required number $\binom{m}{t+1} (t+1)$ of responses can be obtained by using $v = \binom{m}{t+1}$ treatment sets S_α , where the sets S_1, S_2, \dots, S_v form an irreducible BIB design of all combinations of $(t+1)$ cultivars selected from the m cultivars and then noting the response on each of the $(t+1)$ cultivars in the mixture treatment sets S_α . Such designs are minimal designs where a minimal treatment design is one in which the number of responses for the cultivars in mixtures is smallest for estimating the parametric contrasts of interest.

A solution of the normal equations for the unknown parameters for estimating the required contrasts can be easily obtained, and will be given for $t = 2$ in the next section.

3. A SOLUTION OF PARAMETERS FOR TREATMENT DESIGNS OF SECTION 2

Consider an experiment with m cultivars in which the experimenter is interested in drawing inferences up to second order specific mixing abilities. As noted in the last section, the minimal treatment design consists of $v = \binom{m}{3}$ treatments S_1, S_2, \dots, S_v is made up of all triples of the m cultivars and obtaining responses for each of the 3 cultivars in each mixture. However, an experimenter may be interested in using blends of n cultivars, $n \geq 3$. The treatments can be laid out in an appropriate experiment design. Let $\bar{y}_{i_j}(S_\alpha)$ be the mean response of the i_j cultivar used in the mixture S_α from an orthogonal experiment design. (One adjusts $\bar{y}_{i_j}(S_\alpha)$ for blocks if a non-orthogonal experiment design is used.) We give the results here using $\bar{y}_{i_j}(S_\alpha)$ from an orthogonal experiment design for general n and specialize the results for $n=3$.

As a consequence of the conditions or restrictions imposed on the parameters, the following is a solution for the parameters which can be used to estimate the contrasts of τ_i 's, $\gamma_{i(j)}$'s, and $\gamma_{i(j,k)}$'s which are of interest to the experimenter:

$$\hat{\tau}_i = \frac{1}{\binom{m-1}{n-1}} \sum_{\substack{\alpha \\ i \in S_\alpha}} \bar{y}_{i_j}(S_\alpha) - \bar{y},$$

$$\hat{\gamma}_{i(j)} = \frac{1}{\binom{m-1}{n-2}} \left[\binom{m-n}{\alpha} \sum_{i,j \in S_\alpha} \bar{y}_{i_j}(S_\alpha) - (n-1) \sum_{\substack{\alpha \\ i \in S_\alpha \\ j \notin S_\alpha}} \bar{y}_{i_j}(S_\alpha) \right],$$

and

$$\hat{\gamma}_{i(jk)} = \frac{1}{s} \left[\sum_{\alpha} \bar{y}_{i(S_{\alpha})} - \binom{m-3}{n-3} (\bar{y} + \hat{\tau}_i) - \binom{m-4}{n-3} \{ \hat{\gamma}_{i(j)} + \hat{\gamma}_{i(k)} \} \right],$$

where

$$\bar{y} = \sum_{i, \alpha} \bar{y}_{i(S_{\alpha})} / \left\{ \binom{m}{n} n \right\}$$

and

$$s = \binom{m-3}{n-3} - 2 \binom{m-4}{n-4} + \binom{m-5}{n-5}.$$

When $n=3$ the above simplify to:

$$\hat{\tau}_i = \frac{1}{\binom{m-1}{2}} \sum_{i \in S_{\alpha}} \bar{y}_{i(S_{\alpha})} - \bar{y},$$

$$\hat{\gamma}_{i(j)} = \frac{1}{(m-1)(m-3)} \left[\binom{m-3}{\alpha} \sum_{i, j \in S_{\alpha}} \bar{y}_{i(S_{\alpha})} - 2 \sum_{\substack{\alpha \\ i \in S_{\alpha} \\ j \notin S_{\alpha}}} \bar{y}_{i(S_{\alpha})} \right],$$

and

$$\begin{aligned} \hat{\gamma}_{i(j,k)} = \frac{1}{(m-2)(m-3)} & \left[\binom{m-3}{\alpha} \sum_{i, j, k \in S_{\alpha}} \bar{y}_{i(S_{\alpha})} \right. \\ & - \binom{m-4}{\alpha} \sum_{\substack{i, j \in S_{\alpha} \\ k \notin S_{\alpha}}} \bar{y}_{i(S_{\alpha})} - \binom{m-4}{\alpha} \sum_{\substack{i, k \in S_{\alpha} \\ j \notin S_{\alpha}}} \bar{y}_{i(S_{\alpha})} \\ & \left. + 2 \sum_{\substack{\alpha \\ i \in S_{\alpha} \\ j, k \notin S_{\alpha}}} \bar{y}_{i(S_{\alpha})} \right], \end{aligned}$$

where

$$\bar{y} = \sum_{i,\alpha} \bar{y}_{i(S_\alpha)} / \left\{ \binom{m}{3} 3 \right\}$$

is the overall mean of the means.

If the means of the cultivar blends are based on r replications and if σ^2 is the error variance for the responses in the original orthogonal design

$$\text{Var}(\hat{\tau}_i - \hat{\tau}_{i'}) = 2\sigma^2 / \left\{ \binom{m-1}{2} r \right\}, \quad i \neq i',$$

$$\text{Var}(\hat{\gamma}_{i(j)} - \hat{\gamma}_{i(j')}) = 2\sigma^2 / \{(m-3)r\}, \quad j \neq j',$$

$$\text{Var}(\hat{\gamma}_{i(j,k)} - \hat{\gamma}_{i(j,k')}) = 2(m-4)\sigma^2 / \{(m-3)r\}, \quad k \neq k',$$

and

$$\text{Var}(\hat{\gamma}_{i(jk)} - \hat{\gamma}_{i(j'k')}) = 2(m-5)\sigma^2 / \{r(m-3)\}, \quad (j,k) \neq (j',k').$$

Expressions for the solutions of parameters can be similarly obtained for other designs of this type, when the interest is in drawing inferences about higher order specific mixing abilities. An analysis of variance can be obtained by standard methods.

4. MODEL AND MINIMAL DESIGNS WITH RESPONSE VARIABLE \bar{y}_{S_α}

Subject to the terminology introduced in Section 1, if \bar{y}_{S_α} is the mean of the mixture treatment $S_\alpha = \{i_1, i_2, \dots, i_n\}$ based on the orthogonal experiment design used,

$$\begin{aligned} E(\bar{y}_{S_\alpha}) = & n\mu + \sum_{j=1}^n \tau_{i_j}^* + \sum_{j=1}^n \delta_{i_j}^* + \sum_{\substack{j,k=1 \\ j \neq k}}^n \gamma_{i_j(i_k)} + \sum_{\substack{j,k,l=1 \\ j \neq k \neq l \neq j}}^n \gamma_{i_j(i_k, i_l)} + \dots \\ & + \gamma_{i_1(i_2, \dots, i_n)} + \gamma_{i_2(i_1, \dots, i_n)} + \dots + \gamma_{i_n(i_1, \dots, i_{n-1})} \end{aligned}$$

$$= \mu^* + \sum_{j=1}^n \tau_{i_j} + \sum_{\substack{j,k=1 \\ j < k}}^n \lambda_{i_j i_k} + \sum_{\substack{j,k,\ell=1 \\ j < k < \ell}}^n \lambda_{i_j i_k i_\ell} + \dots + \lambda_{i_1 i_2, \dots, i_n},$$

where

$$\begin{aligned} \lambda_{i_j i_k} &= \gamma_{i_j(i_k)} + \gamma_{i_k(i_j)}, \quad \lambda_{i_j i_k i_\ell} = \gamma_{i_j(i_k, i_\ell)} \\ &+ \gamma_{i_k(i_j, i_\ell)} + \gamma_{i_\ell(i_j, i_k)}, \quad \dots \quad \lambda_{i_1, i_2, \dots, i_n} = \gamma_{i_1(i_2, \dots, i_n)} \\ &+ \gamma_{i_2(i_1, \dots, i_n)} + \dots \gamma_{i_n(i_1, i_2, \dots, i_{n-1})}. \end{aligned}$$

For convenience, we may put $\lambda_{i_k i_j} = \lambda_{i_j i_k}$, $\lambda_{i_j i_k i_\ell} = \lambda_{i_j i_\ell i_k} = \lambda_{i_k i_j i_\ell} = \lambda_{i_k i_\ell i_j} = \lambda_{i_\ell i_j i_k} = \lambda_{i_\ell i_k i_j}$, etc. The parameter $\lambda_{i_j i_k}$ behaves like an interaction term for the i_j and i_k cultivars; but in essence it is the sum of combining abilities of i_j cultivar with i_k and that of i_k cultivar with i_j . A similar interpretation can be given for the other λ parameters. In the same vein as in Section 2, we may call $\lambda_{i_j i_k}$ as the first order specific mixing ability of i_j and i_k cultivars, $\lambda_{i_j i_k i_\ell}$ as the second order specific mixing ability of i_j , i_k and i_ℓ cultivars, \dots , $\lambda_{i_1, i_2, \dots, i_n}$ as the n^{th} order specific mixing ability of i_1, i_2, \dots, i_n cultivars.

As in Section 2, the effects can be reparameterized, if necessary, and the following conditions or restrictions may be imposed on the parameters:

$$\sum_{i=1}^m \tau_i = 0,$$

$$\sum_{\substack{j=1 \\ j \neq i_k}}^m \lambda_{i_j i_k} = 0, \text{ for a given } i_k,$$

$$\sum_{\substack{k=1 \\ k \neq i_j}}^m \lambda_{i_j i_k} = 0, \text{ for a given } i_j,$$

$$\sum_{\substack{j=1 \\ i_j \neq i_k, i_l}}^m \lambda_{i_j i_k i_l} = 0, \text{ for given } i_k \text{ and } i_l, i_k \neq i_l,$$

$$\sum_{\substack{k=1 \\ i_k \neq i_j, i_l}}^m \lambda_{i_j i_k i_l} = 0, \text{ for given } i_j \text{ and } i_l, i_j \neq i_l,$$

$$\sum_{\substack{l=1 \\ i_l \neq i_j, i_k}}^m \lambda_{i_j i_k i_l} = 0, \text{ for given } i_j \text{ and } i_k, i_j \neq i_k,$$

$$\sum_{\substack{l=1 \\ i_l \neq i_2, \dots, i_n}}^m \lambda_{i_1 i_2, \dots, i_n} = 0, \quad \sum_{\substack{l=1 \\ i_l \neq i_1, i_3, \dots, i_n}}^m \lambda_{i_1 i_2, \dots, i_n} = 0,$$

$$\dots \sum_{\substack{n=1 \\ i_n \neq i_1, i_2, \dots, i_{n-1}}}^m \lambda_{i_1 i_2, \dots, i_n} = 0.$$

For any k , the number of k^{th} order specific combining ability parameters $\lambda_{i_1 i_2 \dots i_{k+1}}$ is $\binom{m}{k+1}$ and the number of restrictions imposed is $\binom{m}{k}$. Thus the number of independent k^{th} order specific combining ability parameters is $\binom{m}{k+1} - \binom{m}{k} = \binom{m}{k} \frac{(m-2k-1)}{k+1}$. If the experimenter is interested in drawing inferences about the τ_i and at most t^{th} order specific mixing abilities of the cultivars using mixtures of the same size with one response for each mixture, the number of responses needed is

$$1 + (m-1) + \binom{m}{1} \frac{(m-3)}{2} + \binom{m}{2} \frac{(m-5)}{3} + \dots + \binom{m}{t} \frac{(m-2t-1)}{t+1} = \binom{m}{t+1}.$$

Thus using m cultivars one can draw inferences on at most $\{(m-1)/2\}^{\text{th}}$ order specific mixing abilities. On the other hand, if an experimenter is interested in estimating t^{th} order specific mixing abilities, the number of cultivars to be used in the experiment m should satisfy $m > 2t+1$. Note that in Section 2, $m > 2t$, whereas here we need $m > 2t+1$. The required number $\binom{m}{t+1}$ of responses can be obtained by using $v = \binom{m}{t+1}$ treatments S_α , where the sets S_1, S_2, \dots, S_v form an irreducible BIB design of all combinations of $(t+1)$ cultivars selected from the m cultivars and then noting the response on each of the v treatments S_α . Such designs are minimal designs for estimating all the parametric contrasts of interest. A solution of the normal equations for the unknown parameters to estimate the required contrasts can be easily obtained and will be given for $t = 2$ in the next section.

5. A SOLUTION FOR PARAMETERS FOR THE TREATMENT DESIGNS OF SECTION 4

Consider an experiment with m cultivars on which the experimenter is interested in drawing inferences up to second order specific mixing abilities as noted in Section 4; the treatment design consists of $v = \binom{m}{3}$ treatments S_1, S_2, \dots, S_n consisting of all triples of the m cultivars and noting the responses on the treatment mixtures. The treatments can be laid out in an appropriate experiment design. Let \bar{y}_{S_α} be the mean response of the S_α treatment mixture when the experiment design is an orthogonal one. One may replace \bar{y}_{S_α} by adjusting it for block effects if a non-orthogonal experiment design is used. We give the results here using \bar{y}_{S_α} .

It can be easily verified that the following is a solution of the parameters which can be used to estimate the contrasts of interest of τ_i s, λ_{ij} s, and λ_{ijk} s, when $n(\geq 3)$ cultivars are used in each S_α :

$$\hat{\tau}_i = \left\{ \sum_{i \in S_\alpha} \bar{y}_{S_\alpha} - \binom{m-1}{n-1} \bar{y} \right\} / \binom{m-2}{n-1} ,$$

$$\hat{\lambda}_{ij} = \left\{ \sum_{i,j \in S_\alpha} \bar{y}_{S_\alpha} - \binom{m-2}{n-2} \bar{y} - \binom{m-3}{n-2} (\hat{\tau}_i + \hat{\tau}_j) \right\} / w ,$$

$$\hat{\lambda}_{ijk} = \left\{ \sum_{i,j,k \in S_\alpha} \bar{y}_{S_\alpha} - \binom{m-3}{n-3} \bar{y} - \binom{m-4}{n-3} (\tau_i + \tau_j + \tau_k) - s(\lambda_{ij} + \lambda_{ik} + \lambda_{jk}) \right\} / u ,$$

where

$$\bar{y} = \sum_{\alpha} \bar{y}_{S_\alpha} / \binom{M}{N} , \text{ s is given previously,}$$

$$w = \binom{m-2}{n-2} - 2 \binom{m-3}{n-3} + \binom{m-4}{n-4} ,$$

and

$$u = \binom{m-3}{n-3} - 3 \binom{m-4}{n-4} + \frac{3}{2} \binom{m-5}{n-5} - \frac{1}{2} \binom{m-6}{n-6} .$$

When $n=3$, the above reduces to

$$\tau_i = \frac{2}{(m-2)(m-3)} \left(\sum_{i \in S_\alpha} \bar{y}_{S_\alpha} - \binom{m-1}{2} \bar{y} \right) ,$$

$$\hat{\lambda}_{ij} = \left\{ \sum_{i,j \in S_\alpha} \bar{y}_{S_\alpha} - (m-2) \bar{y} - (m-3)(\hat{\tau}_i + \hat{\tau}_j) \right\} / (m-4) ,$$

$$\hat{\lambda}_{ijk} = \sum_{i,j,k \in S_\alpha} \bar{y}_{S_\alpha} - \bar{y} - \hat{\tau}_i - \hat{\tau}_j - \hat{\tau}_k - \hat{\lambda}_{ij} - \hat{\lambda}_{ik} - \hat{\lambda}_{jk} ,$$

where $\bar{y} = \sum_{\alpha} \bar{y}_{S_{\alpha}} / \binom{m}{3}$ is the overall mean of means.

If σ^2 is the variance of the experimental unit total response in blends of cultivars and if the replications are used for each blend in the orthogonal design

$$V(\hat{\tau}_i - \hat{\tau}_{i'}) = 4\sigma^2 / \{r(m-2)(m-3)\}, \quad i \neq i'$$

$$V(\hat{\lambda}_{ij} - \hat{\lambda}_{ij'}) = 2(m-3) \sigma^2 / \{r(m-2)(m-4)\}, \quad i \neq j'$$

$$W(\hat{\lambda}_{ijk} - \hat{\lambda}_{ijk'}) = \frac{2\sigma^2}{r} \left\{ \frac{(m-5)^2}{(m-3)^2} + \frac{2(m-5)^2}{(m-3)^2(m-4)^2} + \frac{4}{(m-3)^2(m-4)^2} \right\}, \quad k \neq k'.$$

Expressions for the solutions of parameters can be similarly obtained for other designs of this type, when the interest is in drawing inferences about higher order specific mixing abilities. An analysis of variance can be obtained by standard methods.

6. APPLICATIONS AND CONCLUDING REMARKS

Several experiments of the type described above have been conducted and are awaiting statistical analyses as given above. Two of these are described below. In the first one the experimenter, Steven Kauffka, Cornell University, was interested in biomass production of six species (cultivars) in mixtures of three. There were 20 such mixtures. His interest centered on general, first order, and second order mixing effects. The experiment design was a randomized complete block design. In a second experiment he used a balanced incomplete block design.

The models and analyses discussed here are not confined to intercropping experiments only. For a second application we describe a study on preference ratings for eight soft drinks in mixtures (groups) of four using a doubly balanced incomplete block treatment design of 14 blocks. The study was conducted by Raghavarao and Wiley (1986). Their interest centered on first order specific mixing (competing) effects of one soft drink on another on preference ratings by individual users of soft drinks.

These results are also useful in studying the effects of one question on another in a survey design using the block total procedure of Raghavarao and Federer (1979). Many other applications could be made.

Unequal numbers of cultivars (items) in a mixture may also be used. Hall (1975) described some procedures for such situations. Note that solutions for $\tau_i = \delta_i + \tau_i^*$ were obtained here. To obtain solutions for δ_i and τ_i^* individually it is necessary to include uniblends (monocultures).

Literature on some of these aspects of statistical analyses for mixture experiments is not extensive or inclusive. Some results may be found in Free and Wilson (1964), Jensen and Federer (1965), Eberhart and Gardner (1966), Smith *et al.* (1974), Hall (1975), Federer (1979), Federer *et al.* (1976, 1982), and Raghavarao and Federer (1979). They consider specific cases. A more general presentation, as given here, is required for statistical analyses of mixture experiments currently being conducted.

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